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A Mathematical Modeling Approach to University Course Planning

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INTRODUCTION

University course scheduling problems are very challenging and highly constrained real-world problems. Many published papers have investigated these kinds of problems, but to the author's knowledge, no research has been done from the student's perspective or the department's long-term course planning perspective. We consider two problems related to university course planning. In the student's course planning problem (SCPP), a student needs to design a course plan that allows him/her to graduate in a timely manner. In the department course planning problem (DCPP), an academic department needs to decide which courses to offer in which semester in order to facilitate student's timely graduation. Three mathematical models for these problems are developed, coded in C++, and solved with IBM ILOG CPLEX. Experiments on small, medium, and large real-world and fictitious instances show promising results.

PROBLEM DESCRIPTION

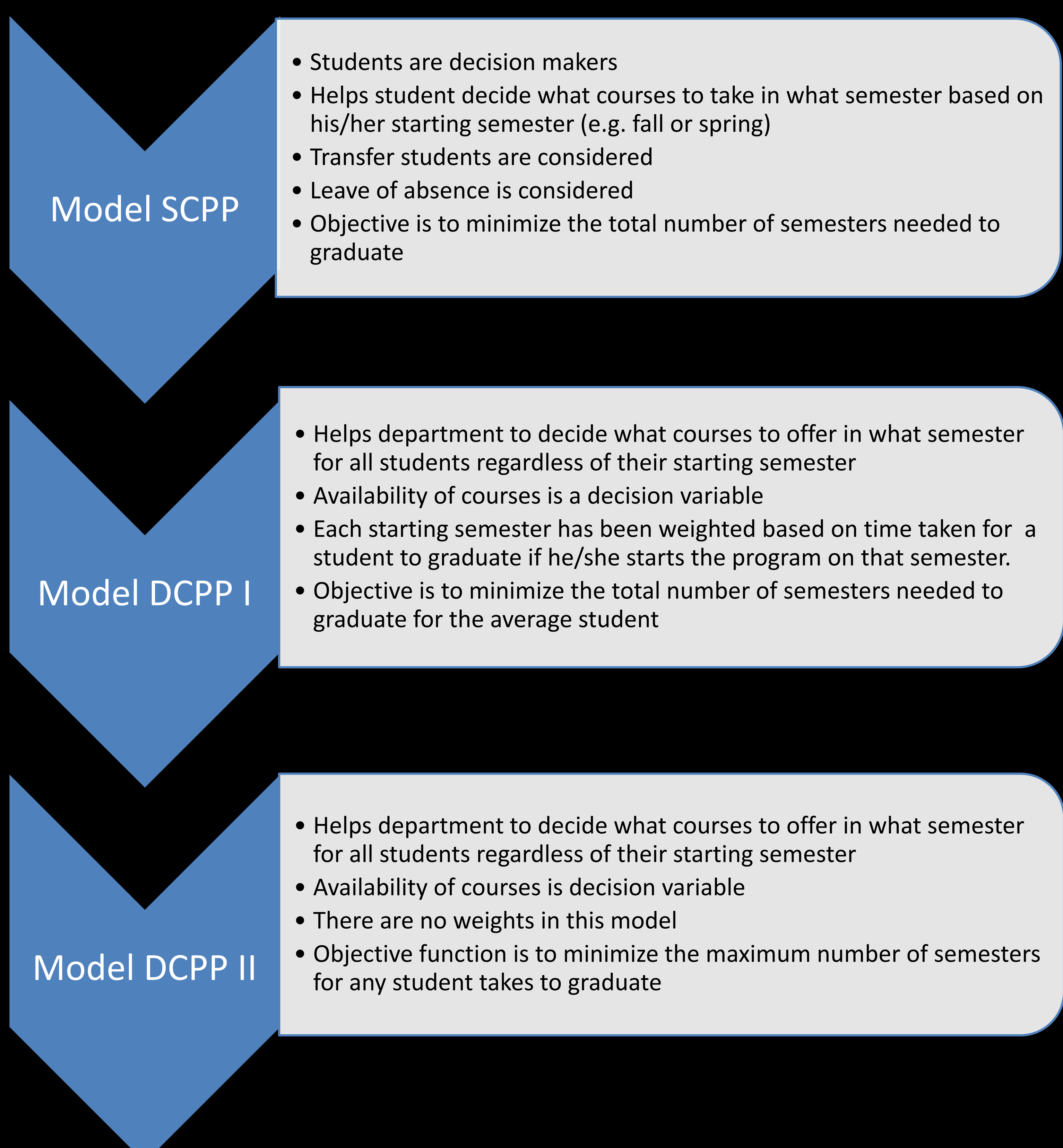

Figure 1: Main features in the mathematical models

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METHODOLOGY:

The main methodology used in this research is mathematical optimization. Three integer programming models are developed to solve defined problems.

Table 1: Parameters and indices used in models

Indices	
c, d	Course (c, d = 1, 2, …., C)
s, t	Semester (s, t = 1, 2, …… S)
N	Number of sessions in each academic year (n = 1, 2……. N) (e.g. fall and spring then N = 2)
Parameters	
C	Number of courses (e.g. 40)
S	Number of semesters (e.g. 10)
N	Number of sessions (e.g. 2)
P_{cd}	= 1 if course c is a pre-req for course d. = 0 otherwise (binary)
C_{cd}	= 1 if course c is a co-req for course d. = 0 otherwise (binary)
J_c	= 1 if junior standing is required for course c = 0 otherwise (binary)
S_c	= 1 if senior standing is required for course c = 0 otherwise
O_{cn}	= 1 if course c is offered in session n. = 0 otherwise (binary)
R_c	= 1 if course c is required for graduation. = 0 otherwise (binary)
Start	session when student is planning to start (1, 2, ……., N)
T_c	= 1 if course c is a technical elective course = 0 otherwise (binary)
TM_c	= 1 if course c is a technical elective course in the major = 0 otherwise (binary)
T	Number of technical elective courses needed for graduation (e.g. 4)
TM	Number of technical elective courses needed in the major for graduation (e.g. 2)
$Rand_c$	Auxiliary parameter that is random real number in range of [0, 0.001). It is used in the second part of the objective function to generate different optimal solutions.
Max	Maximum number of courses student can take per semester
A_c	= 1 if course c has already been taken (e.q. by a transfer student) = 0 otherwise
LA	= 2-S if leave of absence taken during that semester = 0 if no leave of absence takes
Dc	= 1 if course c is offered by department = 0 otherwise (binary)
Allowance	Maximum number of courses that can be offered in an academic year
W_n	Weight for the time taken for student who starts in session n to graduate.
O_{cn}	= 1 if course c is offered in session n. (In model SCPP) = 0 otherwise (binary)
Decision Variables	
X_{ncs}	= 1 if student who started in session n takes course c in semester s. = 0 otherwise (binary)
Y_{ns}	= 1 if student who started in session n takes at least 1 course in semester s. = 0 otherwise. (binary)
D_{ns}	Number of courses completed by the beginning of semester s for students who started in session n
O_{cn}	= 1 if course c is offered in session n. (In model DCPP I and DCPP II) = 0 otherwise (binary)
K	greatest number of semesters for any student takes to graduate

SCPP

Objective function

$$\text{Min} \sum_{s=1}^{S} Y_s + \sum_{s=1}^{S}\sum_{c=1}^{C}(s)(X_{cs})(Rand_c)$$

Constraints

1. $Y_{s+1} \le Y_s$ for all $s \le S-1$
2. $X_{cs} \le Y_s$ for all c and s
3. $A_c + \sum_{s=1}^{S} X_{cs} \ge R_c$ for all c
4. $\sum_{s=1}^{S} X_{cs} \le 1$ for all c
5. $\sum_{c=1}^{C} X_{cs} \le Max$ for all s
6. $\sum_{s=1}^{S} X_{ds} \le \sum_{s=1}^{S} X_{cs} + A_c$ for all (c, d) such that $P_{cd} = 1$
$(\sum_{s=1}^{S}(s)(X_{cs})) + 1 \le (\sum_{s=1}^{S}(s)(X_{ds})) + (S+1)(1 - \sum_{s=1}^{S} X_{ds})$
7. $\sum_{s=1}^{S} X_{ds} \le \sum_{s=1}^{S} X_{cs} + A_c$ for all (c, d) such that $C_{cd} = 1$
$(\sum_{s=1}^{S}(s)(X_{cs})) \le (\sum_{s=1}^{S}(s)(X_{ds})) + (S)(1 - \sum_{s=1}^{S} X_{ds})$
8. $D_s = \sum_{t=1}^{s-1}\sum_{c=1}^{C} X_{ct} + \sum_{c=1}^{C} A_c$ for all $s \ge 2$
$D_s \ge (20)(X_{cs})$ for all c and s such that $J_c = 1$
$D_s \ge (30)(X_{cs})$ for all c and s such that $S_c = 1$
9. $X_{cs} \le O_{c,\,([(s-1)+(Start-1)] \bmod N)+1}$ for all c and s
10. $[\sum_{c=1}^{C}(A_c + \sum_{s=1}^{S} X_{cs})(TM_c)] \ge TM$
$[\sum_{c=1}^{C}(A_c + \sum_{s=1}^{S} X_{cs})(T_c)] \ge T$
11. $Y_{LA} = 1$ If $LA \ne 0$
$\sum_{c=1}^{C} X_{cLA} = 0$

DCPP I

Objective function

$$\text{Min} \sum_{n=1}^{N}\sum_{s=1}^{S}(W_n)(Y_{ns}) + \sum_{s=1}^{S}\sum_{c=1}^{C}(s)(X_{cs})(Rand_c)$$

Constraints

1. $Y_{(n)(s+1)} \le Y_{ns}$ for all n and $s \le S-1$
2. $X_{ncs} \le Y_{ns}$ for all n, c and s
3. $\sum_{s=1}^{S} X_{ncs} \ge R_c$ for all c an n
4. $\sum_{s=1}^{S} X_{ncs} \le 1$ for all c and n
5. $\sum_{c=1}^{C} X_{ncs} \le Max$ for all s and n
6. $\sum_{s=1}^{S} X_{nds} \le \sum_{s=1}^{S} X_{ncs}$ for all (n, c, d) such that $P_{cd} = 1$
$(\sum_{s=1}^{S}(s)(X_{ncs})) + 1 \le (\sum_{s=1}^{S}(s)(X_{nds})) + (S+1)(1 - \sum_{s=1}^{S} X_{nds})$
7. $\sum_{s=1}^{S} X_{nds} \le \sum_{s=1}^{S} X_{ncs}$ for all (n, c, d) such that $C_{cd} = 1$
$(\sum_{s=1}^{S}(s)(X_{ncs})) \le (\sum_{s=1}^{S}(s)(X_{nds})) + (S)(1 - \sum_{s=1}^{S} X_{nds})$
8. $D_s = \sum_{t=1}^{s-1}\sum_{c=1}^{C} X_{nct}$ for all $s \ge 2$ and n
$D_s \ge (20)(X_{ncs})$ for all n,c and s such that $J_c = 1$
$D_s \ge (30)(X_{ncs})$ for all n,c and s such that $S_c = 1$
9. $X_{ncs} \le O_{c,\,([(s-1)+(n-1)] \bmod N)+1}$ for all n,c,s
10. $\sum_{c=1}^{C}\sum_{s=1}^{S}(X_{ncs})(TM_c) \ge TM$ for all n
$\sum_{c=1}^{C}\sum_{s=1}^{S}(X_{ncs})(T_c) \ge T$ for all n
11. $\sum_{n=1}^{N}\sum_{c=1}^{C}(D_c)(O_{cn}) \le Allowance$

DCPP II

Objective function

$$\text{Min } K + \sum_{s=1}^{S}\sum_{c=1}^{C}(s)(X_{cs})(Rand_c)$$

Constraints

1. $Y_{(n)(s+1)} \le Y_{ns}$ for all n and $s \le S-1$
2. $X_{ncs} \le Y_{ns}$ for all n, c and s
3. $\sum_{s=1}^{S} X_{ncs} \ge R_c$ for all c an n
4. $\sum_{s=1}^{S} X_{ncs} \le 1$ for all c and n
5. $\sum_{c=1}^{C} X_{ncs} \le Max$ for all s and n
6. $\sum_{s=1}^{S} X_{nds} \le \sum_{s=1}^{S} X_{ncs}$ for all (n, c, d) such that $P_{cd} = 1$
$(\sum_{s=1}^{S}(s)(X_{ncs})) + 1 \le (\sum_{s=1}^{S}(s)(X_{nds})) + (S+1)(1 - \sum_{s=1}^{S} X_{nds})$
7. $\sum_{s=1}^{S} X_{nds} \le \sum_{s=1}^{S} X_{ncs}$ for all (n, c, d) such that $C_{cd} = 1$
$(\sum_{s=1}^{S}(s)(X_{ncs})) \le (\sum_{s=1}^{S}(s)(X_{nds})) + (S)(1 - \sum_{s=1}^{S} X_{nds})$
8. $D_s = \sum_{t=1}^{s-1}\sum_{c=1}^{C} X_{nct}$ for all $s \ge 2$ and n
$D_s \ge (20)(X_{ncs})$ for all n,c and s such that $J_c = 1$
$D_s \ge (30)(X_{ncs})$ for all n,c and s such that $S_c = 1$
9. $X_{ncs} \le O_{c,\,([(s-1)+(n-1)] \bmod N)+1}$ for all n,c,s
10. $\sum_{c=1}^{C}\sum_{s=1}^{S}(X_{ncs})(TM_c) \ge TM$ for all n
$\sum_{c=1}^{C}\sum_{s=1}^{S}(X_{ncs})(T_c) \ge T$ for all n
11. $\sum_{n=1}^{N}\sum_{c=1}^{C}(D_c)(O_{cn}) \le Allowance$
12. $K \ge \sum_{s=1}^{S} Y_{ns}$ for all n

RESULTS

All Models are tested on data from the UW-Milwaukee Department of Industrial Engineering. For students who start in the fall session, take up to 6 courses/semester, take no leave of absence, and have no incoming transfer courses, the minimum number of semesters needed to graduate is 7. For those who start in spring, the minimum number of semesters is 8. Table 2 shows five optimal solutions for the case where a student starts in the fall.

Table 2: Five outputs from model SCPP (starting in fall, 6 courses/semester, no leaves of absence, no incoming transfer credits)

Starting Session 1(fall)					
Semester 1:	EAS 100 Math 116 Chem 102 English 310 Free Elective EAS 001	EAS 100 Math 116 Chem 102 Social Science 1 Social Science 2 Free Elective	EAS 100 EAS 200 Math 116 Chem 102 English 310 Free Elective	EAS 100 IE 111 Math 116 Chem 102 Social Science 2 English 310	IE 111 Math 116 Chem 102 Art Humanities English 310
Semester 2:	IE 111 IE 112 MatlEng 201 Math 231 Chem 104 Humanities	EAS 200 IE 112 MatlEng 201 Math 231 Chem 104 English 310	MatlEng 201 Math 231 Chem 104 Art Social Science 2 EAS 001	EAS 200 IE 112 MatlEng 201 Math 231 Chem 104 EAS 001	EAS 100 IE 112 CompSci 240 MatlEng 201 Math 231 Chem 104
Semester 3:	CompSci 240 IE 350 IE 367 Math 232 Physics 209 Social Science 1	CompSci 240 IE 350 IE 367 Math 232 Physics 209 Humanities	CompSci 240 IE 350 IE 367 Math 232 Physics 209 Humanities	CompSci 240 IE 350 IE 367 Math 232 Physics 209 Free Elective	EAS 200 IE 350 IE 367 Math 232 Physics 209 Social Science 1
Semester 4:	CivEng 201 EE 301 IE 475 IE 575 Math 233 Physics 210	CivEng 201 EE 301 IE 475 IE 575 Math 233 Physics 210	CivEng 201 EE 301 IE 575 Math 233 EE 234 Physics 210	CivEng 201 EE 301 IE 475 Math 233 EE 234 Physics 210	CivEng 201 EE 301 IE 575 Math 233 EE 234 Physics 210
Semester 5:	CivEng 202 IE 360 IE 370 IE 455 IE 580 MechEng 301	CivEng 202 IE 370 IE 455 IE 470 IE 580 EE 234	CivEng 202 IE 370 IE 455 IE 470 IE 580 MechEng 301	CivEng 202 IE 360 IE 370 IE 455 IE 470 IE 580	CivEng 202 IE 360 IE 370 IE 455 IE 470 IE 580
Semester 6:	EAS 200 IE 465 IE 571 EE 234 Social Science 2 IE 405	IE 360 IE 465 IE 571 IE 572 IE 587 MechEng 474	IE 111 IE 360 IE 465 IE 475 IE 571 IE 590	IE 465 IE 571 IE 575 Humanities IE 584 Bus Adm 330	IE 465 IE 475 IE 571 IE 550 IE 572 MechEng 474
Semester 7:	IE 470 IE 485 IE 583 Art IE 584	IE 111 IE 485 IE 583 Art Bus Adm 330	IE 112 IE 485 IE 583 Social Science 1 IE 582	IE 485 IE 583 Art Social Science 1 IE 582	IE 485 IE 583 Social Science 2 Free Elective IE 582

Table 3: Outputs from model SCPP (with leave of absence, no incoming transfer credits, Max=5 and 6)

Start=1 (fall) and Max =6	
LA	#Semesters (including LA)
2	9
3	9
4	9
5	9
6	8
7	9

Start=2 (spring) and Max =6	
LA	#Semesters (including LA)
2	8
3	8
4	8
5	8
6	10
7	8
8	10

Start=1 (fall) and Max = 5	
LA	#Semesters (including LA)
2	10
3	10
4	10
5	10
6	10
7	10
8	10
9	10

Start=2 (spring) and Max = 5	
LA	#Semesters (including LA)
2	10
3	10
4	10
5	10
6	10
7	10
8	10
9	10

Table 3 shows that if *Max*=6 and a student starts the program on fall, leave of absence on any semester except semester 6, will delay graduation for one year but if the student takes the leave of absence on semester 6, the student's graduation will be delayed by one semester Which is important information that academic advisor can use to inform new students. Also, if a student starts the program on spring semester, only leave of absence on semesters 6 and 8 will delay student's graduation (by one year). If Max=5, regardless of starting semester, graduation will be delayed by one semester.

The result of model DCPP I with W_n = [0.9 , 0.8] as an input for weight is 7. In other word, theoretically regardless of the student's starting semester, with above weights, the student will be able to graduate in 7 semesters. The result for model DCPP II for this case study is also 7. It is worth to mention again that in these two models course availability is a decision variable and it is being designed to minimize the objective function.

CONCLUSION

In this research, we developed 3 mathematical models from 2 different views, coded them in C++ and solved them with CPLEX. All three models are tested on data from the UW-Milwaukee Department of Industrial Engineering. In all models it has been assumed that all other departments have infinite resources for offering their courses. All models are generic and can be applied to different cases. As a future study, this research can be extended to develop a model for the school of engineering course planning.

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